Neutral Higgs Boson Production at LC in the CP violating Two Higgs Doublet Model

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Introduction

- General model with two Higgs doublets
  : No discrete symmetry is imposed

  $\rightarrow$ FCNC mediated by neutral Higgs boson
  $\rightarrow$ Spontaneous and explicit CP violation in the Higgs sector

- Neutral Higgs boson production
  - Scalar-pseudoscalar mixing as a CP violating signal
  - Production cross sections are modified in the CP violating two Higgs doublet model at LC
  - Supplementary informations are obtained from $e^- e^+ \rightarrow Z h_i$ and $e^- e^+ \rightarrow h_i h_j$ processes

The Model

- The general Higgs potential with two Higgs doublets

$$
\begin{aligned}
V \quad = \quad & \frac{1}{2}\lambda_1(\phi_1^\dagger\phi_1)^2 + \frac{1}{2}\lambda_2(\phi_2^\dagger\phi_2)^2 + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) \\
& + \frac{1}{2}[\lambda_5(\phi_1^\dagger\phi_2)^2 + H.c.] + [\lambda_6(\phi_1^\dagger\phi_1)(\phi_1^\dagger\phi_2) + \lambda_7(\phi_2^\dagger\phi_2)(\phi_1^\dagger\phi_2) + H.c.] \\
& - m_{11}^2((\phi_1^\dagger\phi_1) - m_{22}^2((\phi_2^\dagger\phi_2) - [m_{12}^2((\phi_1^\dagger\phi_2) + H.c.]
\end{aligned}
$$

- Natural Flavour conservation : Z_2 symmetry imposed

$$\phi_1 \to -\phi_1, \quad \phi_2 \to \phi_2 \qquad \to \qquad \lambda_6 = \lambda_7 = m_{12}^2 = 0$$

$\to$ Tree level FCNC and CP violation are absent.

- Hard violation : $\lambda_6 \neq 0, \quad \lambda_7 \neq 0$

  Soft violation : $m_{12}^2 \neq 0$

- Minimizing the potential at

$$\langle\phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \langle\phi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_2 e^{i\xi} \end{pmatrix},$$

where $\tan\beta = v_2/v_1$, $v^2 = v_1^2 + v_2^2$, we obtain the relation

$$\mathrm{Im}(m_{12}^2 e^{i\xi}) = v_1 v_2 \mathrm{Im}(\lambda_5 e^{2i\xi})$$

- Rephasing invariance under the global transform $\phi_i \to \phi_i e^{i\varphi_i}$ by

$$\lambda_5 \to \lambda_5 e^{-2i(\varphi_2-\varphi_1)}, \quad m_{12}^2 \to m_{12}^2\, e^{-i(\varphi_2-\varphi_1)},$$
$$\xi \to \xi + \varphi_2 - \varphi_1,$$
$$\lambda_i,\ i = 1,2,3,4 \ \text{ and } \ m_{11,22}^2 \ \text{ invariant}$$

$\rightarrow$ We have a freedom to choose $\xi = 0$
indicating no spontaneous CP violation but wholly explicit CP violation.

- The neutral states are defined by

$$\begin{aligned} G^0 &= \sqrt{2}(\mathrm{Im}\ \phi_1^0 \cos\beta + \mathrm{Im}\ \phi_2^0 \sin\beta), \\ A^0 &= \sqrt{2}(-\mathrm{Im}\ \phi_1^0 \sin\beta + \mathrm{Im}\ \phi_2^0 \cos\beta), \\ \varphi_1 &= \sqrt{2}\mathrm{Re}\ \phi_1^0, \\ \varphi_2 &= \sqrt{2}\mathrm{Re}\ \phi_2^0. \end{aligned}$$

- Physical states are defined by the rotation

$$\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = \mathcal{R} \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ A \end{pmatrix} \tag{1}$$

- The mass matrix of neutral Higgs bosons

$$\mathcal{M}^2 = \begin{pmatrix} \mathcal{M}_{11}^2 & \mathcal{M}_{12}^2 & -\frac{1}{2}\mathrm{Im}(\lambda_5)\sin\beta \\ \mathcal{M}_{21}^2 & \mathcal{M}_{22}^2 & -\frac{1}{2}\mathrm{Im}(\lambda_5)\cos\beta \\ -\frac{1}{2}\mathrm{Im}(\lambda_5)\sin\beta & -\frac{1}{2}\mathrm{Im}(\lambda_5)\cos\beta & \mathcal{M}_{33}^2 \end{pmatrix} v^2$$

where

$$\begin{aligned} \mathcal{M}_{11}^2 &= R\sin^2\beta + \lambda_1\cos^2\beta, \\ \mathcal{M}_{22}^2 &= R\cos^2\beta + \lambda_2\sin^2\beta, \\ \mathcal{M}_{12}^2 &= (\lambda_3 + \lambda_4 + \mathrm{Re}\lambda_5 - R)\,\frac{\sin 2\beta}{2}, \\ \mathcal{M}_{33}^2 &= R - \mathrm{Re}\lambda_5, \end{aligned}$$

with $R = \mathrm{Re}(m_{12}^2)/v_1 v_2$.

- Diagonalization of the mass matrix

$$\mathcal{M}_d^2 = \mathcal{R}\mathcal{M}^2\mathcal{R}^\dagger$$

- Parametrizaton of the rotation matrix

$$\begin{aligned}\mathcal{R} &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_c & s_c \\ 0 & -s_c & c_c \end{pmatrix} \begin{pmatrix} c_b & 0 & s_b \\ 0 & 1 & 0 \\ -s_b & 0 & c_b \end{pmatrix} \begin{pmatrix} -s_a & c_a & 0 \\ c_a & s_a & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} -c_b s_a & c_a c_b & s_b \\ c_a c_c + s_a s_b s_c & s_a c_c - c_a s_b s_c & c_b s_c \\ -c_a s_c + s_a s_b c_c & -s_a s_c - c_a s_b c_c & c_b c_c \end{pmatrix},\end{aligned}$$

where $s_{a,b,c} = \sin\theta_{a,b,c}$ and $c_{a,b,c} = \cos\theta_{a,b,c}$.

- The CP-odd state A is mixed with CP-even states φ_1, φ_2

Neutral Higgs Boson Production $e^+e^- \to Zh_i$

- Generalized h_iZZ vertices

$$\begin{aligned} h_1ZZ &\sim \sin(\beta-\alpha)\cos\alpha_b, \\ h_2ZZ &\sim \cos(\beta-\alpha)\cos\alpha_c - \sin(\beta-\alpha)\sin\alpha_b\sin\alpha_c, \\ h_3ZZ &\sim -\cos(\beta-\alpha)\sin\alpha_c - \sin(\beta-\alpha)\sin\alpha_b\cos\alpha_c. \end{aligned}$$

- The cross sections for $e^+e^- \to h_iZ$ processes

$$\sigma(e^+e^- \to h_iZ) = \frac{f_i^2\pi\alpha^2\lambda^{1/2}(\lambda+12sm_Z^2)\left[1+(1-4\sin^2\theta_W)^2\right]}{192s^2\sin^4\theta_W\cos^4\theta_W(s-m_Z^2)^2}$$

where where f_i are the h_iZZ coupling given above

$$\lambda = \lambda(s, m_h^2, m_Z^2)$$

with $\lambda(a,b,c) = (a+b-c)^2 - 4ab$.

Neutral Higgs Boson Production $e^+e^- \to h_ih_j$

- Generalized Zh_ih_j vertices

$$
\begin{aligned}
Zh_1h_3 &\sim \cos(\beta-\alpha)\cos\alpha_c - \sin(\beta-\alpha)\sin\alpha_b\sin\alpha_c, \\
Zh_2h_3 &\sim -\sin(\beta-\alpha)\cos\alpha_b, \\
Zh_1h_2 &\sim \cos(\beta-\alpha)\sin\alpha_c + \sin(\beta-\alpha)\sin\alpha_b\cos\alpha_c.
\end{aligned}
$$

- The cross sections for $e^+e^- \to h_ih_j$ processes

$$
\begin{aligned}
\sigma(e^+e^- \to h_ih_j) &= \frac{g^4}{196\pi\cos^2\theta_W} f_{ij}^2 \left(\frac{8\sin^4\theta_W - 4\sin^2\theta_W + 1}{\cos^2\theta_W}\right) \\
&\times \frac{\kappa^3}{\sqrt{s}\,[(s-m_Z^2)^2 + \Gamma_Z^2 m_Z^2]}
\end{aligned}
$$

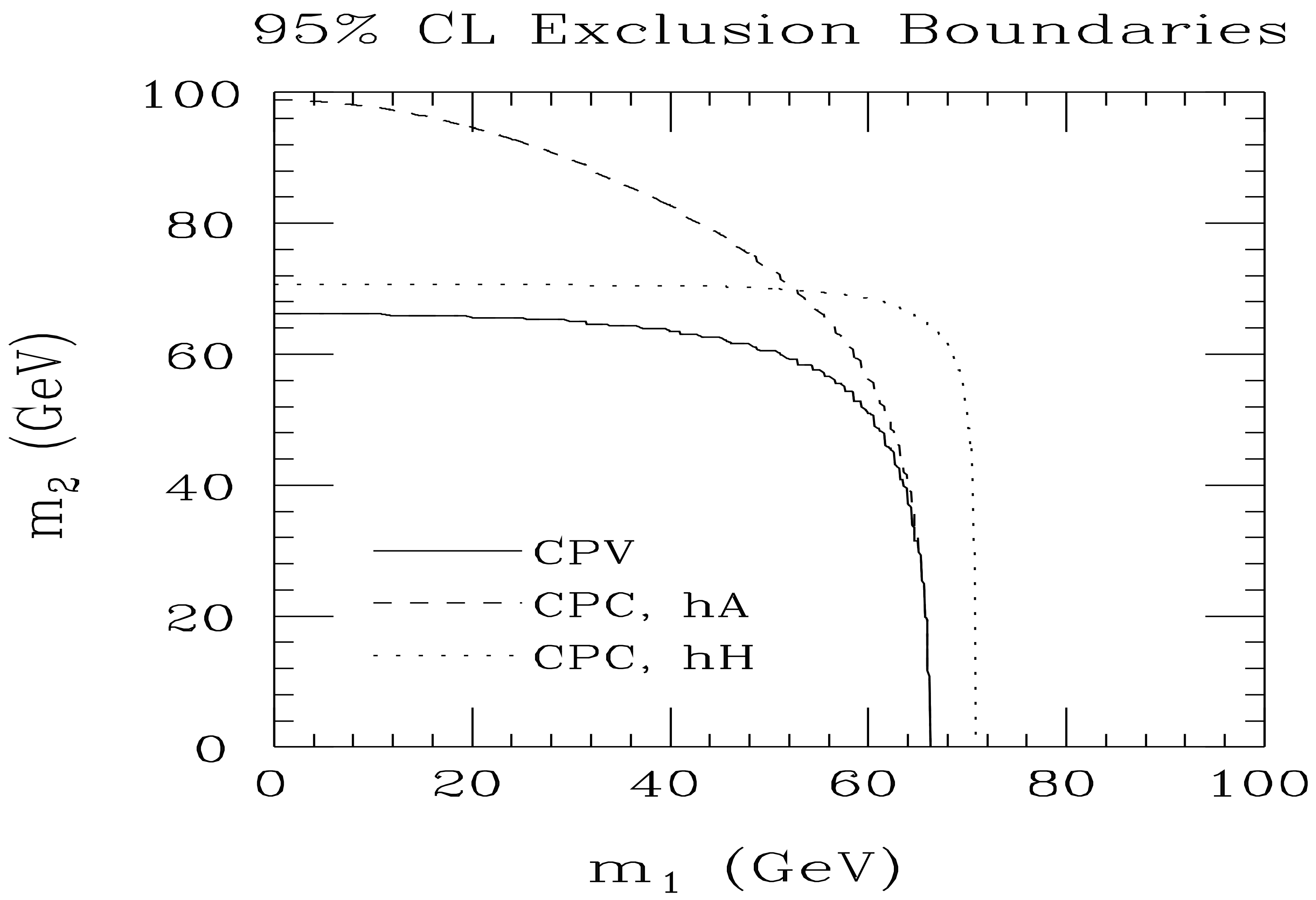
where f_{ij} are the h_ih_jZ coupling given above and the kinematic factor

$$
\kappa^2 = \frac{\lambda(s, m_{h_i}^2, m_{h_j}^2)}{4s}
$$

A few Sum Rules for Higgs couplings

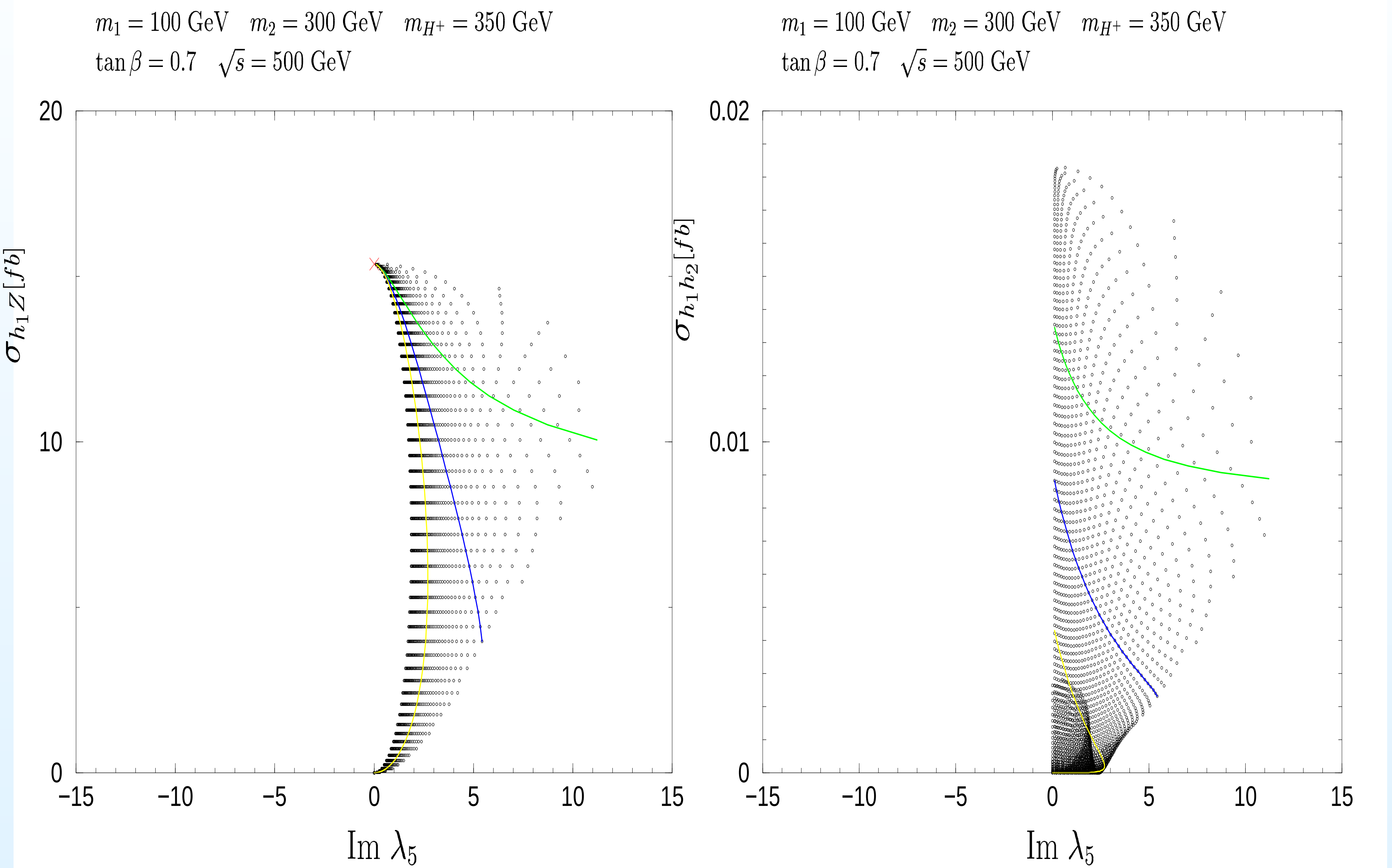
- Consequences of trigonometric relation given above
- $(h1ZZ)^2 + (h_2ZZ)^2 + (h_3ZZ)^2 = 1$
- $(Zh_1h_2)^2 + (Zh_2h_3)^2 + (Zh_3h_1)^2 = 1$
- $(h_iZZ)^2 + (h_jZZ)^2 + (Zh_ih_i)^2 = 1, i \neq j$
- $e^-e^+ \to Zh_i$ and $e^-e^+ \to h_ih_j$ are supplementary to each other.

Limits from LEP data



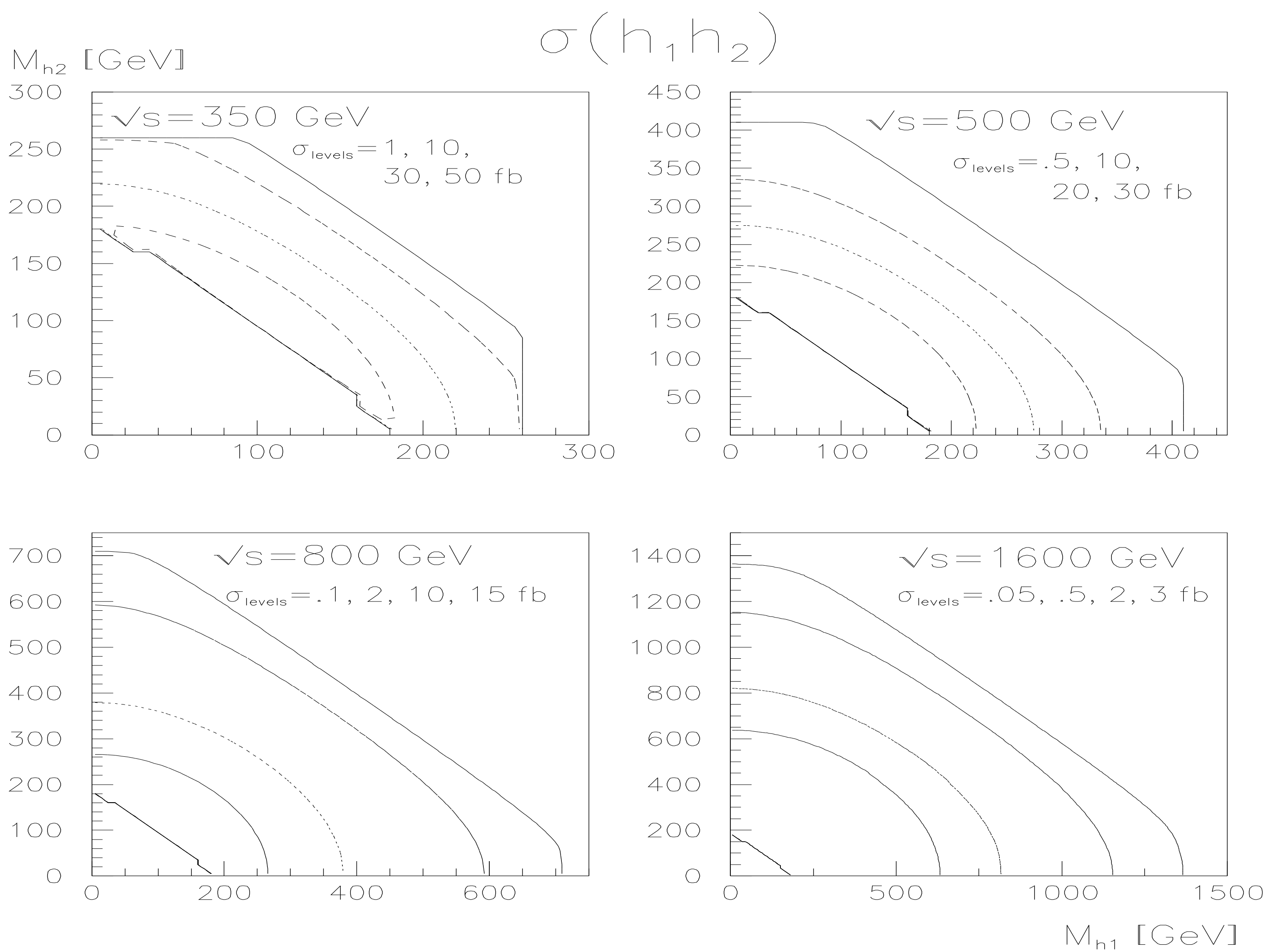
Numerical Analysis

- Inputs : m_1, m_2, $\tan\beta$, $m_\pm$
- variables : $\theta_a \equiv \alpha$, θ_b, θ_c
- Constraints
  : ordering, $m_1 < m_2 < m_3$,
  : perturbativity, $\lambda/4\pi < 1$
- $\beta - \alpha = \pi/6$
  $\theta_c = \pi/6$ (Green), $\theta_c = \pi/4$ (Blue), $\theta_c = \pi/3$ (Yellow)



$m_1 = 100$ GeV $m_2 = 300$ GeV $m_{H^+} = 350$ GeV
$\tan\beta = 0.7$ $\sqrt{s} = 500$ GeV
$\sigma_{h_1 Z}[fb]$
20
10
0
−15
−10
−5
0
5
10
15
Im λ_5
$m_1 = 100$ GeV $m_2 = 300$ GeV $m_{H^+} = 350$ GeV
$\tan\beta = 0.7$ $\sqrt{s} = 500$ GeV
$\sigma_{h_1 h_2}[fb]$
0.02
0.01
0
Im λ_5

Prediction of $\sigma(e^-e^+ \to h_1 h_2)$ at LC



Concluding Remarks

- The 2 Higgs doublet model with CP violation may enhance or reduce the $e^-e^+ \to Zh$ and $e^-e^+ \to h_i h_j$ cross sections compared with those of CP conserving case.
- In the limit of $\sin\theta_c \to 0$ and $\sin\theta_b \to 1$, the ratio of hZZ and HZZ couplings are reversed to that of the CP conserving case and the mixing angle $\alpha (= \theta_a)$ is close to $-\beta$.
- The neutral Higgs boson production shows very sensitive behavior near the CP conserving limit
- The 2 Higgs doublet model with CP violation will be able to be tested at the LC through neutral Higgs boson production.