Well-known unknown: new measurements of the Cabibbo angle

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Outline

- V_{us} and unitarity of CKM matrix
- Ways to extract V_{us}
- Situation before 2004: did we have self-consistent picture ?
- Hints of a problem – K^+ case
- Breakthrough – K_L, K_S case
- Support – hyperons.
- Conclusions

V_{us} and Unitarity check

CKM matrix describes the quark mixing:

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \tag{1}$$

V_{us} is the oldest known mixing element (Cabibbo angle). Yet many exciting developments have happened just this year !

Unitarity of CKM matrix requires:

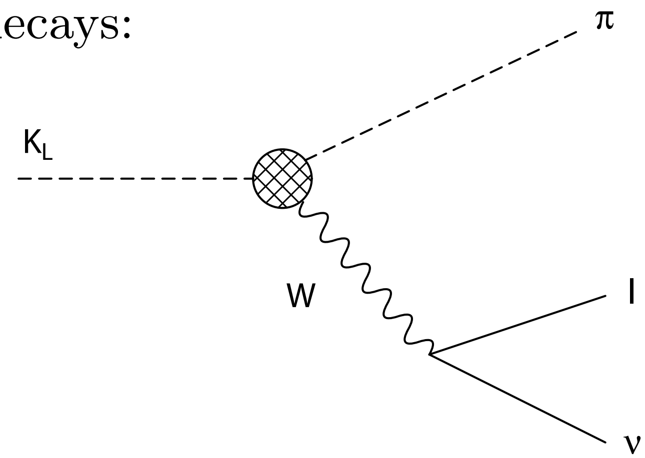
$$1 - (|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2) = \delta = 0 \tag{2}$$

Largest contribution comes from $|V_{ud}|$, next from $|V_{us}|$, negligible from $|V_{ub|}$.

According PDG-02, $\delta = 0.0043 \pm 0.0019$, about 2.2σ deviation from unitarity, with uncertainty from V_{us} of 0.0010.

Methods to extract V_{us}

The most accurate approach to extract V_{us} is to use rate of semileptonic kaon decays:



$$\Gamma_{K\ell 3} = \frac{G_F^2 M_K^5}{192\pi^3} S_{EW}(1+\delta_K^\ell)C^2 \left|V_{us}\right|^2 f_+^2(0) I_K^\ell, \qquad (3)$$

Here:

- S_{EW}, δ_K^ℓ – universal short- and mode dependent long-distance radiative corrections.
- $C = 1$ for K_L and $C = 1/2$ for $K^\pm$.
- $f_+^2(0)$ is calculated in theory form factor value for $t = 0$
- I_K^ℓ are mode and form factor ($f+(t)$ for $Ke3$ and $f_+(t), f_0(t)$ for $K\mu 3$) dependent decay phase space integrals.

Situation before 2004

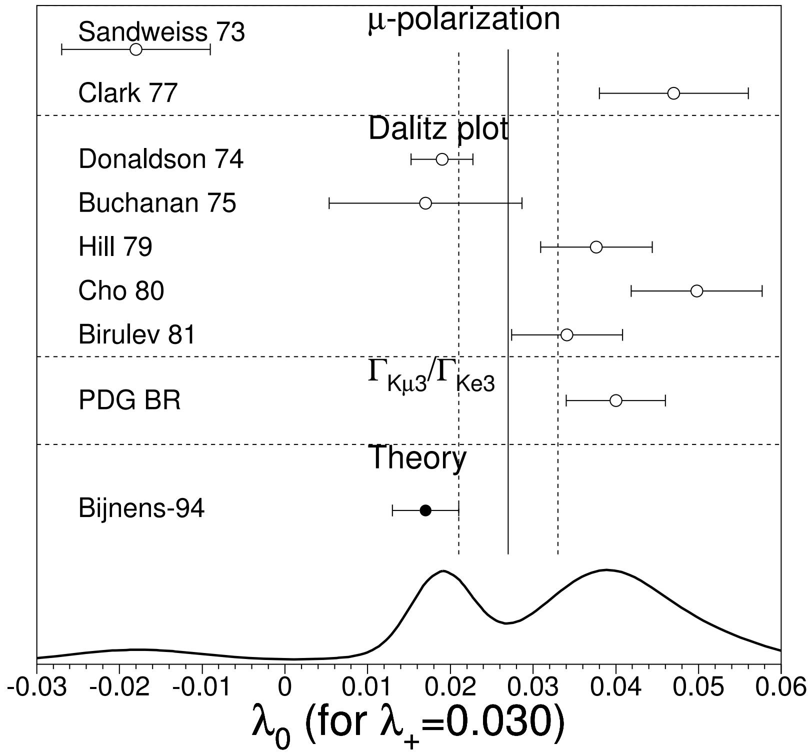
Apart from unitarity problem, V_{us} seemed to be well understood before the new data has arrived:

- Measured with $K_L e3$ ($0.2182 \pm 0.0012_{\rm exp}$), $K^{\pm} e3$ ($0.2208 \pm 0.0016_{\rm exp}$) and Hyperon decays ($0.2176 \pm 0.0026$). The most precise measurement came from $K_L e3$ decays.
- $K_L e3$ branching fraction is extracted from various measurements of 36 different experiments performed between 1967-1995, they show good internal agreement
- $f_+(t)$ form factor is measured by ~ 10 experiments, well described by linear λ^+ term. The value of λ^+ is consistent between $K^{\pm}$ (0.028 ± 0.003) and K_L (0.030 ± 0.002) as well as with theory (chiral QCD) expectations (~ 0.028).
- $f_+(0)$ is calculated by Leutweyler and Roos in 1984, their analysis shows that $K^{\pm} e3$ and $K_L e3$ data are consistent.

The only problem in this picture was BNL E865 determination of V_{us} based on $K^{\pm} e3$ data (PRL **91** 261802, published on 31 Dec 2003) which triggered a lot of new experimental activity.

Consistency check: $Ke3$ vs $K\mu3$

V_{us} measured with $Ke3$ should be equal to V_{us} measured with $K\mu3$ ("lepton universality"). Also, $f_+^{Ke3}(t) = f_+^{K\mu3}(t)$. For a linear parameterization of $f_0(t)$ this allows to extract λ_0 from $Br(K\mu3)/Br(Ke3)$:.



- unsatisfactory experimental situation.
- theory (which is used for $f_+(0)$) largely disagree with BR result

K^+ result from BNL E865

Measurement of $Br(K^+e3)$ based on 70.000 decays normalized to $Br(K^+ \to \pi^+\pi^0)$, $Br(K^+ \to \pi^0\mu^+\nu)$ and $Br(K^+ \to \pi^+\pi^0\pi^0)$.

π^0 is detected using Dalitz $\pi^0 \to e^+e^-\gamma$ decay.

Assuming PDG-02 values for the branching fractions of the normalization modes, using also new calculations of $f+(0)$ and long distance radiative corrections, E865 experiment extracts:

$$V_{us} = 0.2272 \pm 0.0023_{rate} \pm 0.0007_{\lambda+} \pm 0.0018_{f+(0)} \quad (4)$$

With this value, CKM unitarity is satisfied within 1σ.

New results in 2004

After the BNL result, we got much more experimental and theoretical attention to V_{us}.

- New measurements/determinations of semileptonic branching fractions: KTeV (also preliminary by NA48, KLOE)
- New measurements of semileptonic form factors: ISTRA+, KTeV (also preliminary by NA48).
- New (preliminary) measurement of K_L lifetime: KLOE
- New calculations of $f_+(0)$ – chiral QCD, lattice QCD.
- New results for Ξ^0 beta decay

KTeV measurement of K_L branching fractions

KTeV is (was) a fixed target experiment to measure $\Re(\epsilon'/\epsilon)$ with 10^{-4} precision. Since there is no way to tag the kaon, measure all six largest decay modes in terms of five branching fraction ratios and use the constraint that the remaining width is just 0.03%. Use external τ_L to convert branching fractions into partial widths.

The five measured ratios are:

$$\Gamma_{K\mu3}/\Gamma_{Ke3} \equiv \Gamma(K_L \to \pi^\pm\mu^\mp\nu)/\Gamma(K_L \to \pi^\pm e^\mp\nu) \tag{5}$$

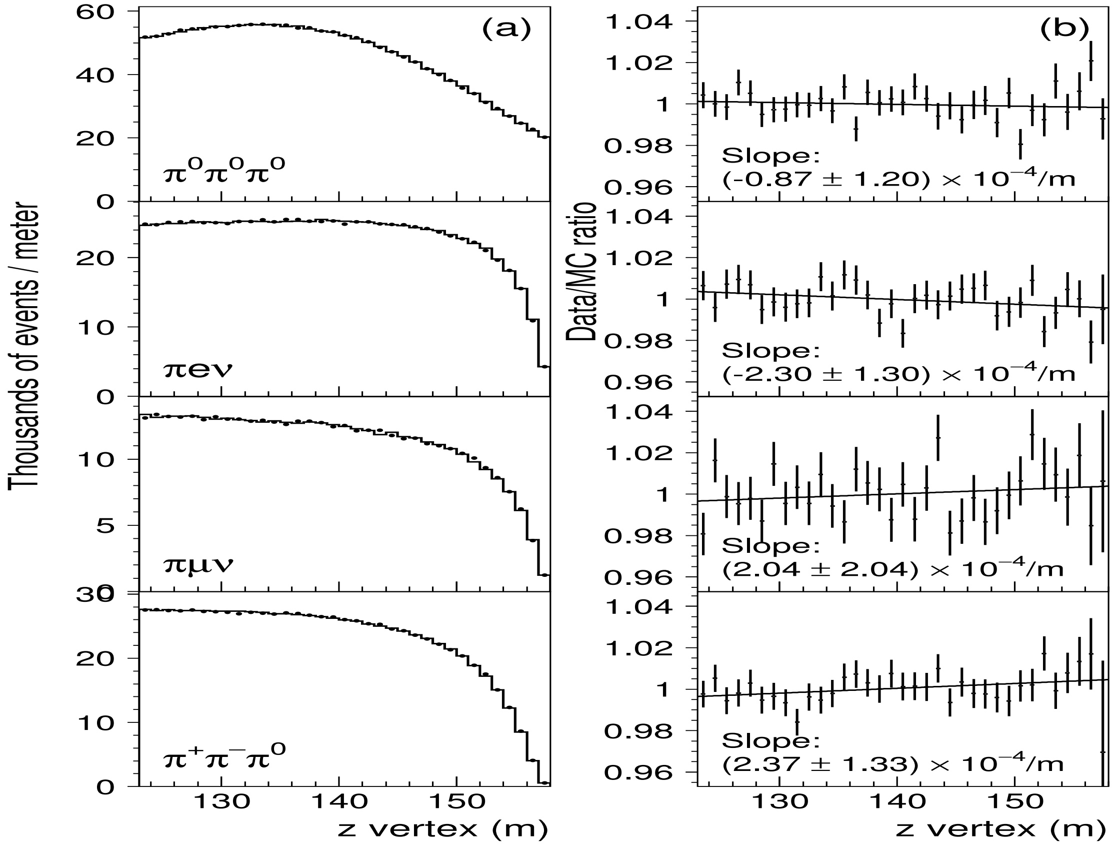
$$\Gamma_{+-0}/\Gamma_{Ke3} \equiv \Gamma(K_L \to \pi^+\pi^-\pi^0)/\Gamma(K_L \to \pi^\pm e^\mp\nu) \tag{6}$$

$$\Gamma_{000}/\Gamma_{Ke3} \equiv \Gamma(K_L \to \pi^0\pi^0\pi^0)/\Gamma(K_L \to \pi^\pm e^\mp\nu) \tag{7}$$

$$\Gamma_{+-}/\Gamma_{Ke3} \equiv \Gamma(K_L \to \pi^+\pi^-)/\Gamma(K_L \to \pi^\pm e^\mp\nu) \tag{8}$$

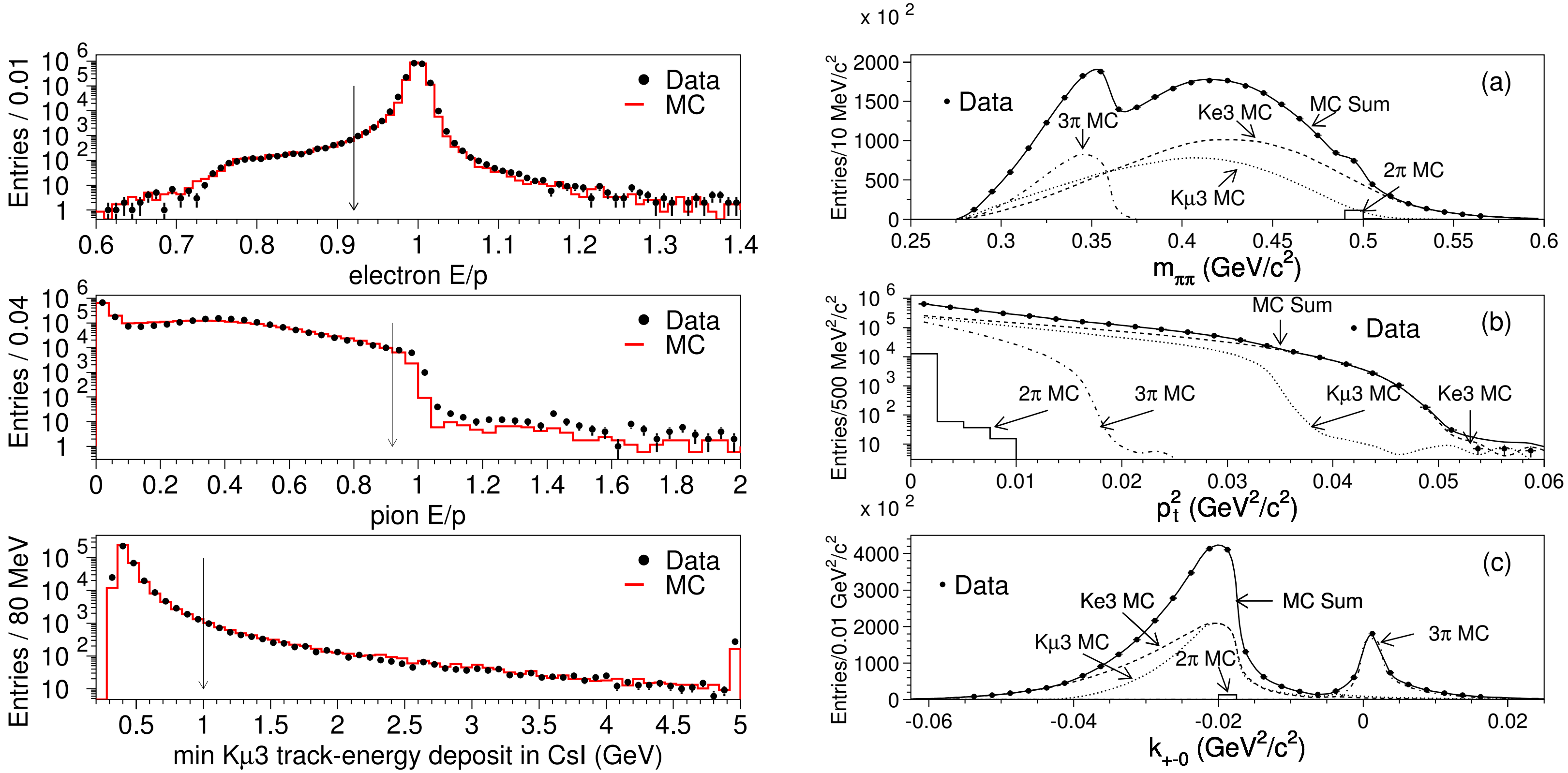
$$\Gamma_{00}/\Gamma_{000} \equiv \Gamma(K_L \to \pi^0\pi^0)/\Gamma(K_L \to \pi^0\pi^0\pi^0), \tag{9}$$

KTeV: Acceptance vs Z



- Acceptance is different for different modes but well described by MC
- Special effort to minimize effects from different particle types (e.g. μ vs π). For example, μ system is not used in the main $K\mu3$ analysis and π^0 decay products are ignored for $\pi^+\pi^-\pi^0$.

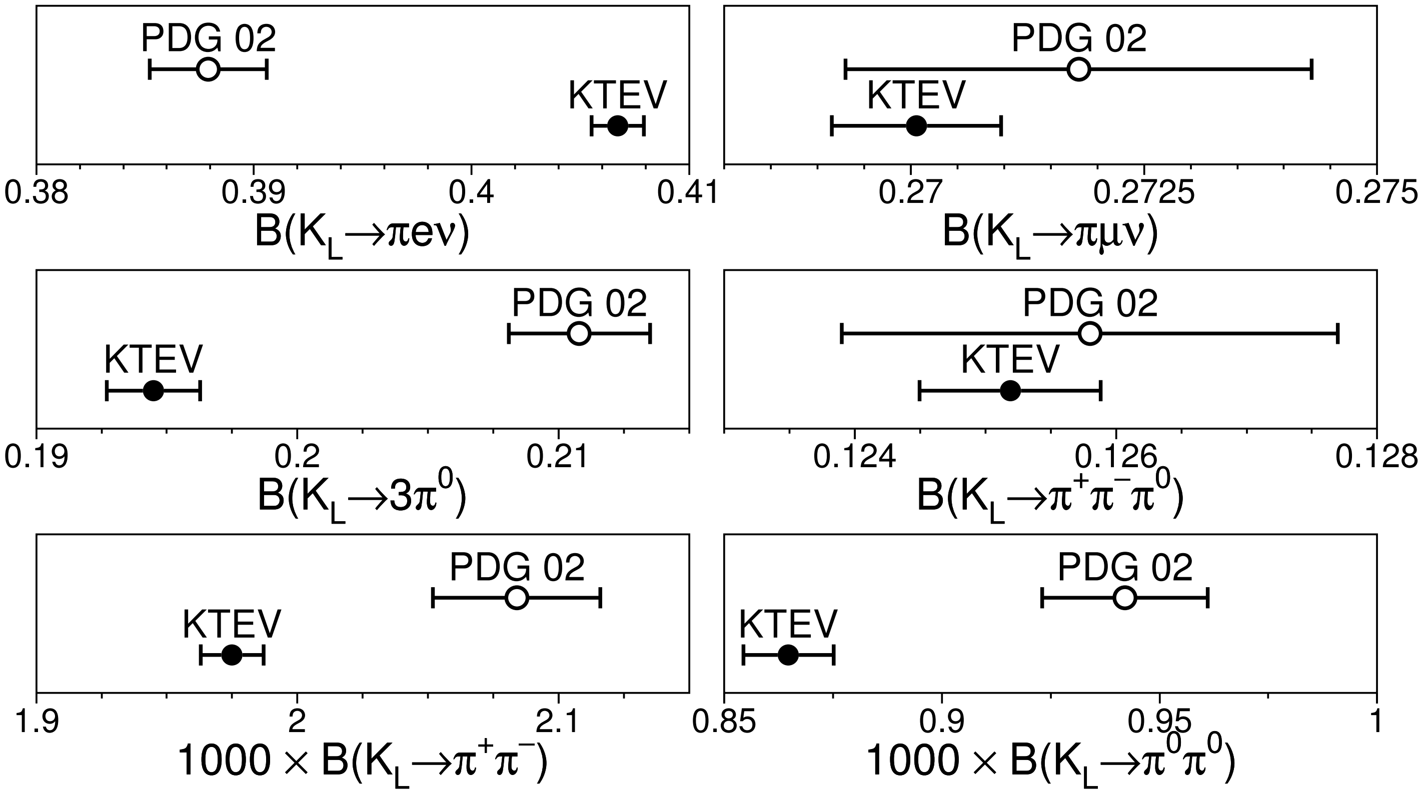
KTeV: Reconstruction of the charged modes



Different charged modes are distinguished from each other using CsI calorimeter energy response (left) and kinematic requirements (right).

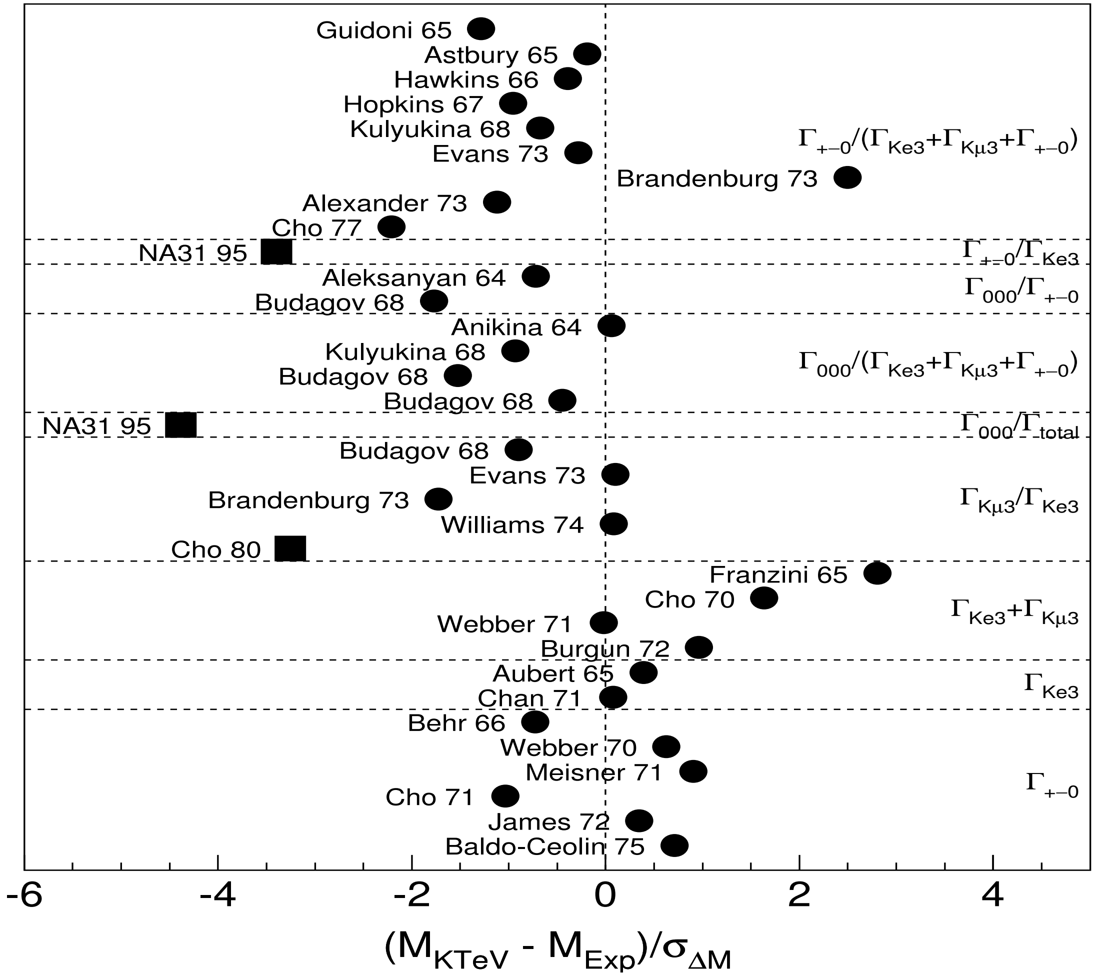
The background for each charged mode is $\leq 0.1\%$.

KTeV results for K_L Branching Fractions



Large change compared to PDG for 4 out of 6 decay modes. In particular, $Ke3$ is about 5% higher. But $K\mu 3$ is consistent with older values.

KTeV vs old experiments

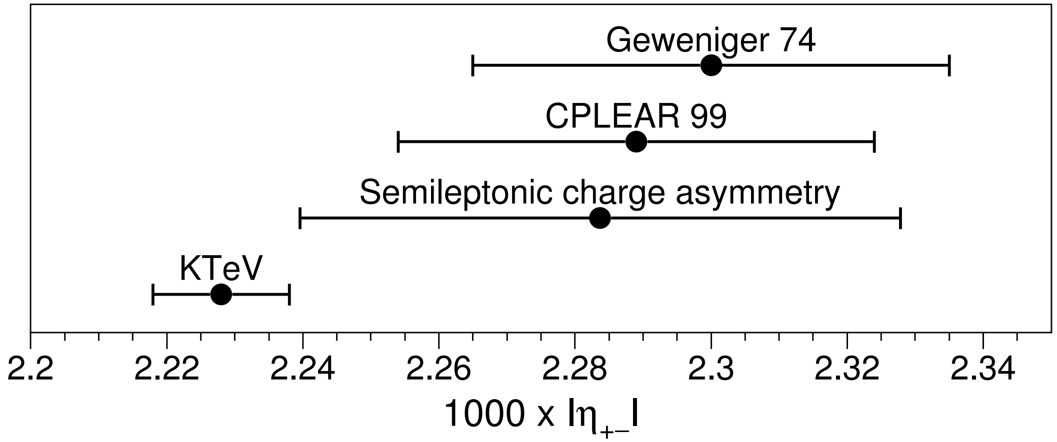


- For all experiments: $\chi^2/dof = 83/34$
- Excluding Cho80, NA31: $\chi^2/dof = 42/31$

Another problem: η_{+-}

Using the measured $K_L \to \pi\pi$ branching fractions, external values of τ_S (KTeV, NA48) and $\tau_L = 5.15 \pm 0.04$ (PDG02), and correcting for small effects of $\Re(\epsilon'/\epsilon)$ and K_S semileptonic branching fraction one obtains η_{+-}

$$\begin{aligned} \eta_{+-} &= \frac{\tau_S}{\tau_L} \frac{B_L(\pi^+\pi^-) + B_L(\pi^0\pi^0)\left[1 + 6\Re(\epsilon'/\epsilon)\right]}{1 - B_S(Kl3)} \\ &= (2.228 \pm 0.010) \times 10^{-3} \end{aligned} \quad (10)$$

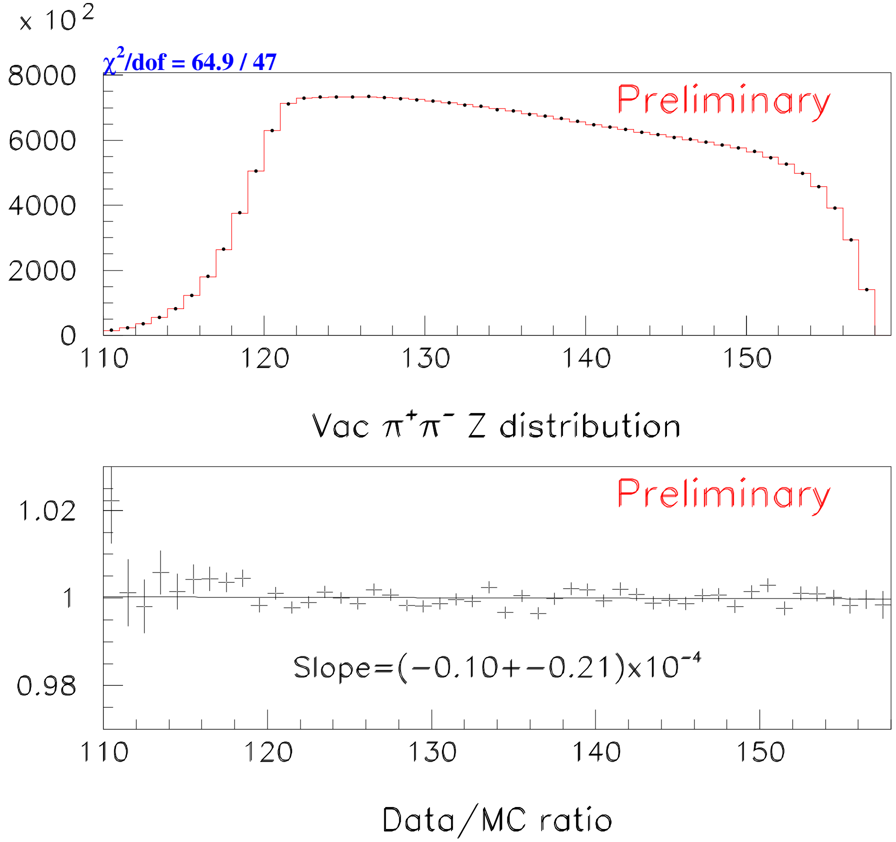


- Most of the error for KTeV is from external τ_L uncertainty
- Geweniger-74 and CPLEAR-99 are $K_L - K_S$ interference based measurements, depend on τ_S, corrected to new τ_S.

From another talk: status of $\Re(\epsilon'/\epsilon)$

NA48 + KTeV-97 $\rightarrow$ $\Re(\epsilon'/\epsilon) = (16.7 \pm 2.3) \times 10^{-4}$.

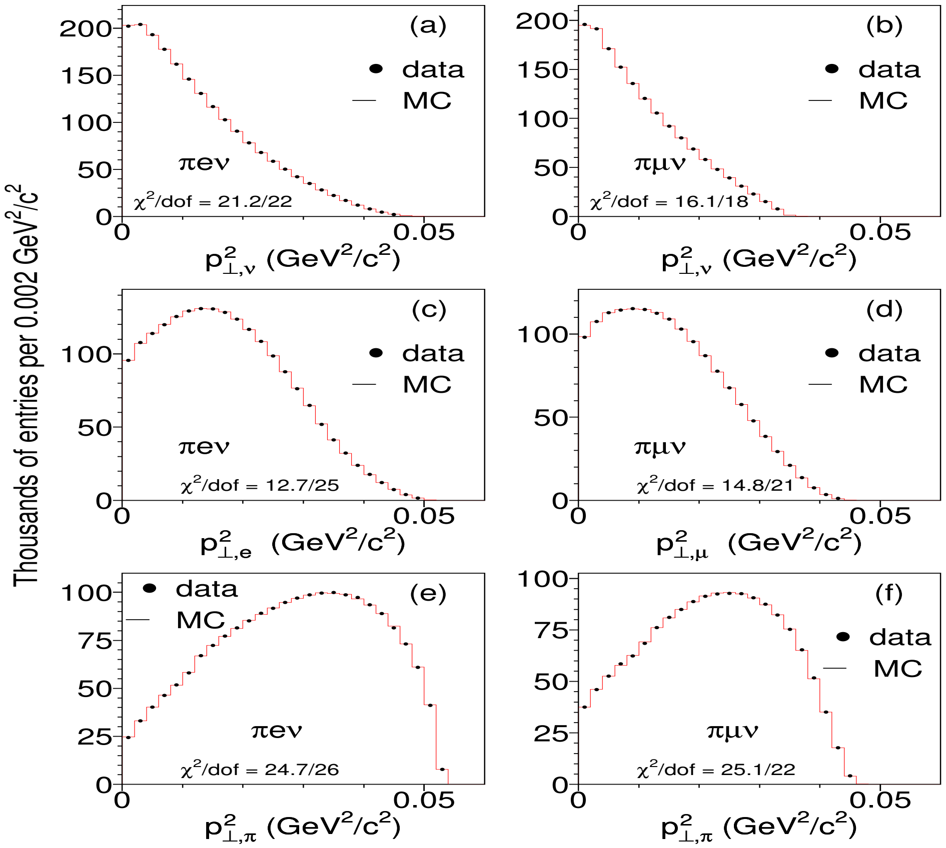
KTeV analysis of 99 data is still in progress:



- $\pi^+\pi^-$ analysis is essentially finalized
- $\pi^0\pi^0$ analysis: tuning CsI MC.

KTeV measurement of semileptonic form factors

Since kaon energy is unknown (2-fold ambiguity) use boost invariant transverse-t determined using $p_\perp$ of the particles.



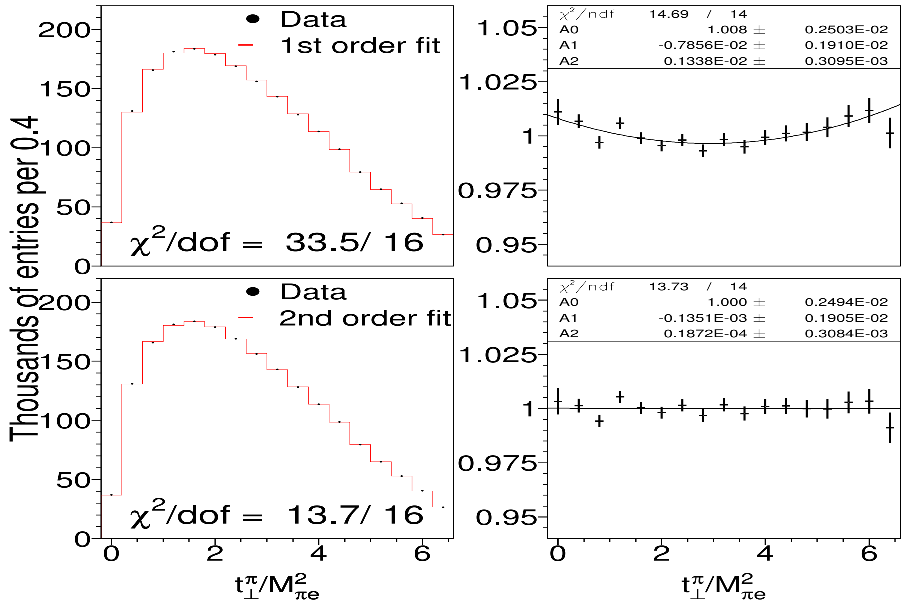
→ good agreement btw data and MC. MC study shows that $t_\perp$-method to extract FF is only about 15% less precise statistically compared to ideal t-based extraction.

Form factors: non-linear term

Parameterization of the form factors:

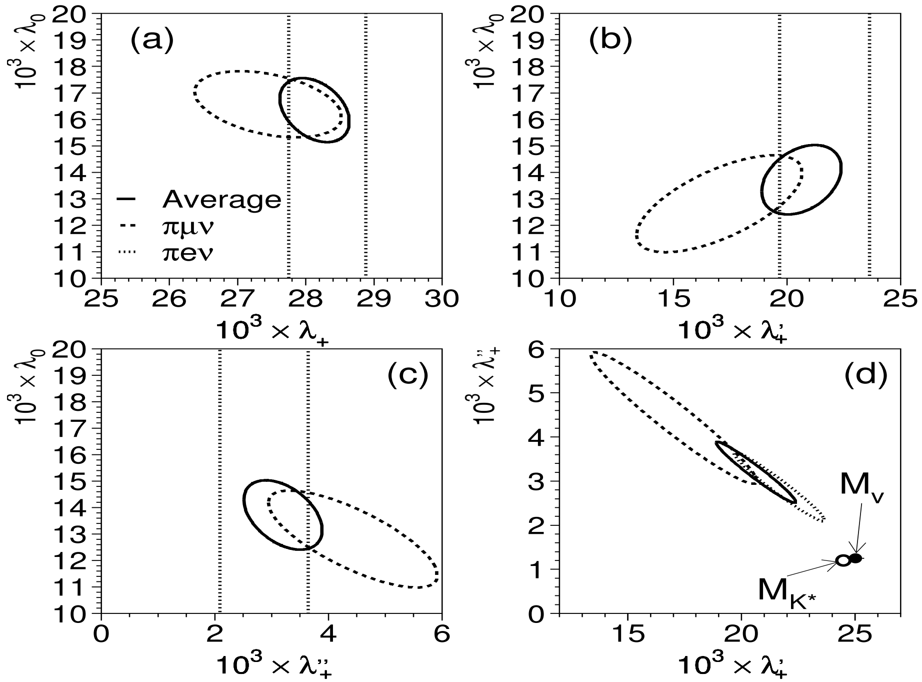
$$
\begin{aligned}
f_+(t) &= f_+(0) \times \left[1 + \lambda'_+ \frac{t}{M_\pi^2} + \frac{1}{2}\lambda''_+ \frac{t^2}{M_\pi^4}\right] \\
f_0(t) &= f_+(0) \times \left[1 + \lambda'_0 \frac{t}{M_\pi^2}\right]
\end{aligned}
\tag{11}
$$

KTeV sees improvement in the fit to $t_\perp$ distribution using the quadratic parameterization for $f_+(t)$:.



→ the second order fit changes I_K integrals by about -1%

Form factor results



KTeV result is consistent with ISTRA+ result for K^+

	λ'_+	λ''_+	λ_0 (for $\lambda_+ = 0.0277$)
	$\times 10^{-3}$		
KTeV	20.64 ± 1.75	3.20 ± 0.69	16.5 ± 1.1
ISTRA+	23.24 ± 1.55	1.68 ± 0.82	18.3 ± 1.1

KTeV check: lepton universality

V_{us} measured with $Ke3$ and $K\mu3$ should be the same – lepton universality. More directly, the ratio of the Fermi coupling constants for electrons and muons must be the same:

$$\left(\frac{G_F^\mu}{G_F^e}\right)^2 = \left[\frac{\Gamma(K_L \to \pi^\pm \mu^\mp \nu)}{\Gamma(K_L \to \pi^\pm e^\mp \nu)}\right] \Big/ \left(\frac{1+\delta_K^\mu}{1+\delta_K^e} \cdot \frac{I_K^\mu}{I_K^e}\right) \tag{12}$$

- Theoretical uncertainties in $f_+(0)$ cancel for this ratio
- "Matching scale" uncertainties for δ_K^ℓ are reduced: $(1+\delta_K^\mu)/(1+\delta_K^e) = 1.0058 \pm 0.0010$
- Uncertainties for the "rate" measurement of $\Gamma(K_L \to \pi^\pm \mu^\mp \nu)/\Gamma(K_L \to \pi^\pm e^\mp \nu) = 0.6640 \pm 0.0026$ differ vs the "shape" measurement of the form factors.
- Ratio of $I_K^\mu / I_K^e = 0.6622 \pm 0.0018$ has reduced dependence on the form factor parameterization.

$$(G_F^\mu / G_F^e)^2 = 0.9969 \pm 0.0048$$

NA48

NA48 presents new preliminary results for

- Measure $B(K_L \to 3\pi^0) = 0.1966 \pm 0.033$ (normalized to $K_S \to 2\pi^0$) — consistent with KTeV
- $B(K_L e3)/B(K_L \to \text{all 2 track}) = 0.498 \pm 0.004$. Using $B(K_L \to 3\pi^0)$ NA48 determines $B(Ke3) = 0.4010 \pm 0.0045$ — again consistent with KTeV.
- $B(K^\pm e3) = (5.14 \pm 0.06)\%$ (using $K^\pm \to \pi^\pm \pi^0$) as normalization mode — consistent with E865.
- New results for Ξ^0 beta decay (see later)
- Measurement of $K_L e3$ form factor (linear parameterization only) $\lambda_+ = 0.0288 \pm 0.0012$, also in agreement with KTeV (0.0283 ± 0.0006).

KLOE: K_S, K_L, K^+

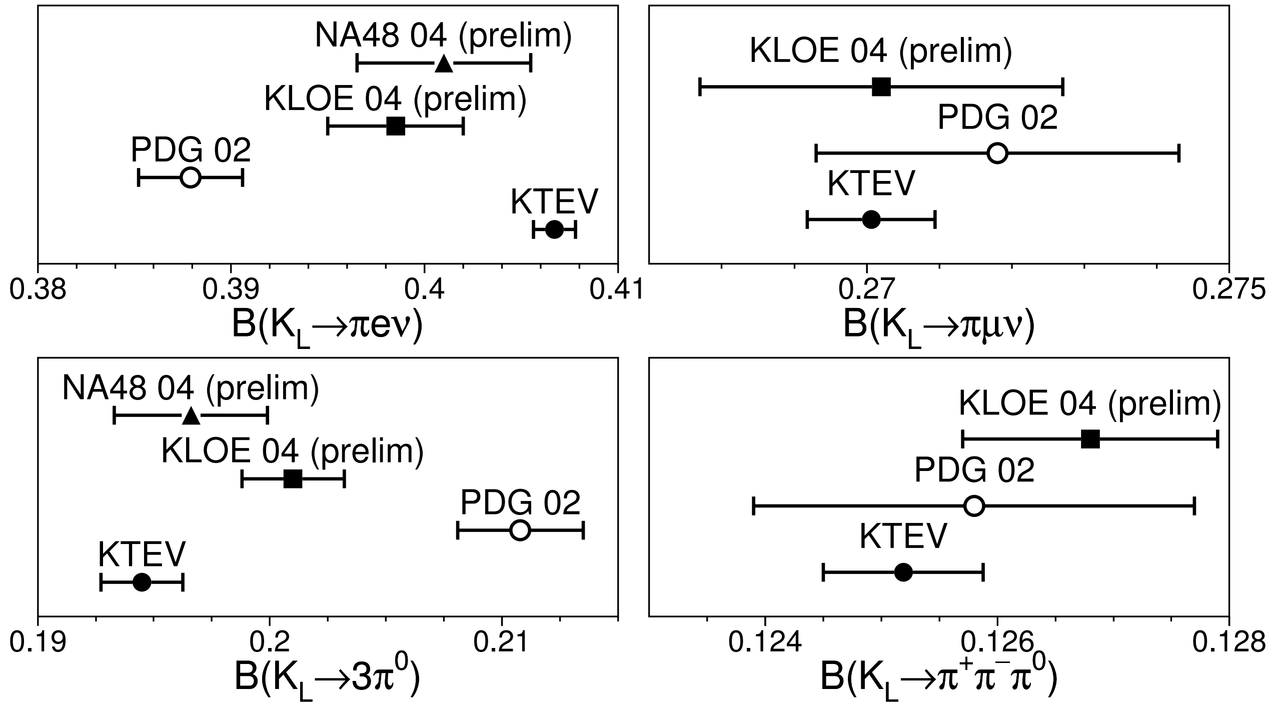
KLOE — ability to tag K_S, K_L and $K^{\pm}$ makes KLOE an ideal experiment to measure branching fractions. A number of new (preliminary) results:

- Precision measurement of
  $Br(K_S \to \pi e \nu) = (7.09 \pm 0.11) \times 10^{-3}$
- Measurement of the four largest K_L branching fractions:

$$\begin{aligned} Br(K_L \to \pi e \nu) &= 0.3985 \pm 0.0035 \\ Br(K_L \to \pi \mu \nu) &= 0.2702 \pm 0.0025 \\ Br(K_L \to \pi^0 \pi^0 \pi^0) &= 0.2010 \pm 0.0022 \\ Br(K_L \to \pi^+ \pi^- \pi^0) &= 0.1268 \pm 0.0011 \end{aligned} \tag{13}$$

- Measurement of the K_L lifetime.

KTeV vs KLOE vs NA48



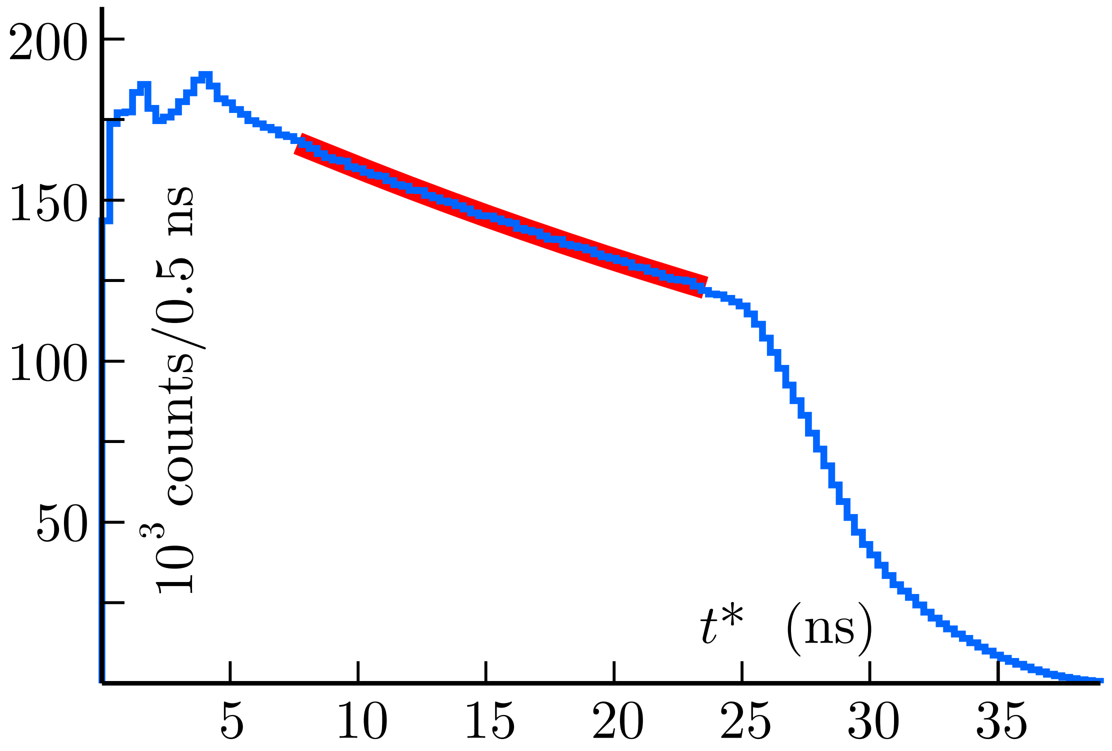
Taking into account correlation between KTeV measurements, $\chi^2/dof = 13.1/6 \rightarrow 4\%$ consistency probability between the new experimental results.

KLOE — lifetime

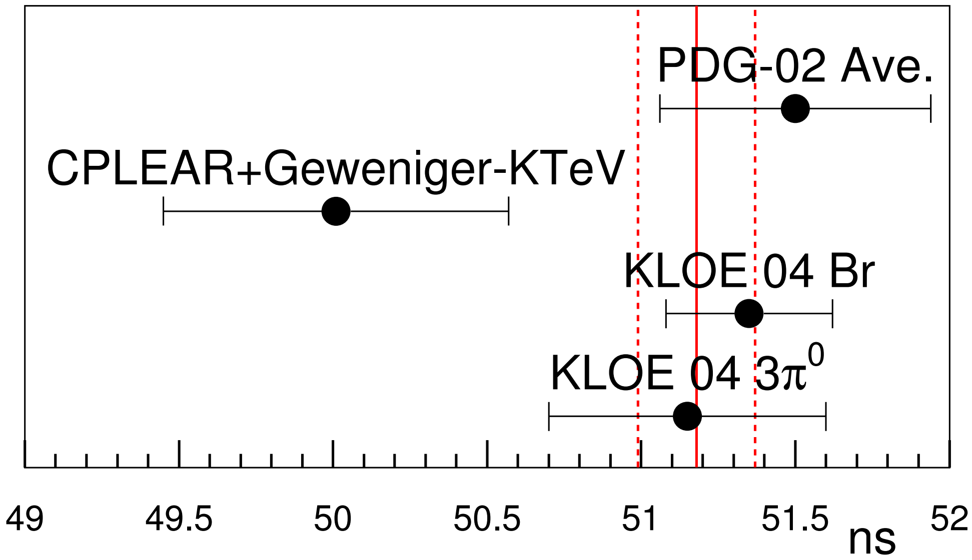
Another very important development from KLOE: new measurements of K_L lifetime ($K^{\pm}$ in progress !)

Two methods:

- Unitarity condition that the four measured modes sum to 99.7% of the total width
- Use $K_L \to 3\pi^0$ decays.



K_L lifetime



Difference between interference and KTeV determination of η_{+-} can be used to determine τ_L.

$$\overline{\tau_L} = (51.18 \pm 0.19)\ \mathrm{ns} \quad (\chi^2/dof = 5.3/3) \tag{14}$$

New Hyperon data from NA48

Ξ^0 beta decay:

$$\Xi^0 \to \Sigma^+ e^- \nu_e \tag{15}$$

$\to$ for the exact $SU(3)$ symmetry identical to neutron β-decay.

NA48 sample from 2002-K_S run – about 6000 events (with 2.4% background) $\to$ much larger compared to the published KTeV sample — 176 events.

$$Br(\Xi^0 \to \Sigma^+ e^- \nu_e) = (2.51 \pm 0.11) \times 10^{-4} \tag{16}$$

Assuming no $SU(3)$ symmetry breaking, $V_{us} = 0.214 \pm 0.030$, consistent with unitarity but the errors are large.

Radiative corrections for $K\ell 3$ decays

Two parts of radiative corrections:

$$\delta_{tot} = S_{EW}(1 + \delta_K) \tag{17}$$

Universal short distance radiative corrections, $S_{EW} = 1.022$, calculated by Sirlin in 1981.

Mode dependent radiative corrections δ_K:

- Originally calculated by Ginsberg in the late 1960s.
- New calculations for $K^0\ell 3$ and $K^{\pm}\ell 3$ using chiral QCD (Cirigliano et al, Bytev et al) and effective theory approach (Andre) – for $K^0 e3$ about 0.5% lower than Ginsberg estimation
- The radiative corrections are included in MC simulation (e.g. **KLOR** program in the case of KTeV)

$\rightarrow$ $\delta_{KLe3} = (1.3 \pm 0.3)\%$, $\delta_{KL\mu 3} = (1.9 \pm 0.3)\%$, the errors include the uncertainty arising from the change of the matching scale.

New theory developments for $f_+(0)$

Original estimate of $f_+(0)$ was made by Leutweyler and Roos (82):

- Complete chiral-QCD calculations up to p^4
- Estimate of p^6 contribution using quark model ($f_4 = -0.016 \pm 0.008$)

$\rightarrow f_+(0) = 0.961 \pm 0.008$ (for K_L).

New estimates:

- New chiral-QCD based calculation of p^6 terms (Bijnens et al, Jamin et al.; $f_4 = -0.002 \pm 0.010$)
- New lattice-QCD (quenched) calculation (Becirevic et al)

The new chiral-QCD calculations tend to return higher values
$\rightarrow f_+(0) = 0.980 \pm 0.010$

The quenched lattice QCD is close to the original estimate:
$\rightarrow f_+(0) = 0.960 \pm 0.009$

Check of the theory: form factors vs experiment

The theoretical estimates of $f_+(0)$ can be checked comparing $f(t)$ predictions vs experiment.

Lattice calculation, "pole model" fit (from the talk of F. Mescia at ICHEP04):

	Theory	KTeV
λ^+	0.025 ± 0.002	0.0250 ± 0.0004
λ^0	0.012 ± 0.002	0.0141 ± 0.0010

$\rightarrow$ good agreement between data and theory.

But chiral-QCD also predicts similar values of λ^+, λ^0.

Putting things together

Experiment:

K_L lifetime is re-measured by KLOE. The new world average value is $\tau_L = 51.18 \pm 0.19$ ns.

$B(K_L \to \pi e \nu)$ is measured by KTeV, KLOE, NA48. All values are higher than PDG-02. KTeV and KLOE are different by $\sim 2\sigma$, NA48 agrees with both.
Need final results from KLOE/NA48 !

$B(K_L \to \pi \mu \nu)$ is measured by KTeV, KLOE and agrees well with PDG-02

$B(K^\pm \to \pi e \nu)$ is measured by NA48, agrees well with E865, significantly higher than PDG-02.

$f_{+,0}(t)$ measured by ISTRA+ for K^+ and KTeV for K_L agree well.

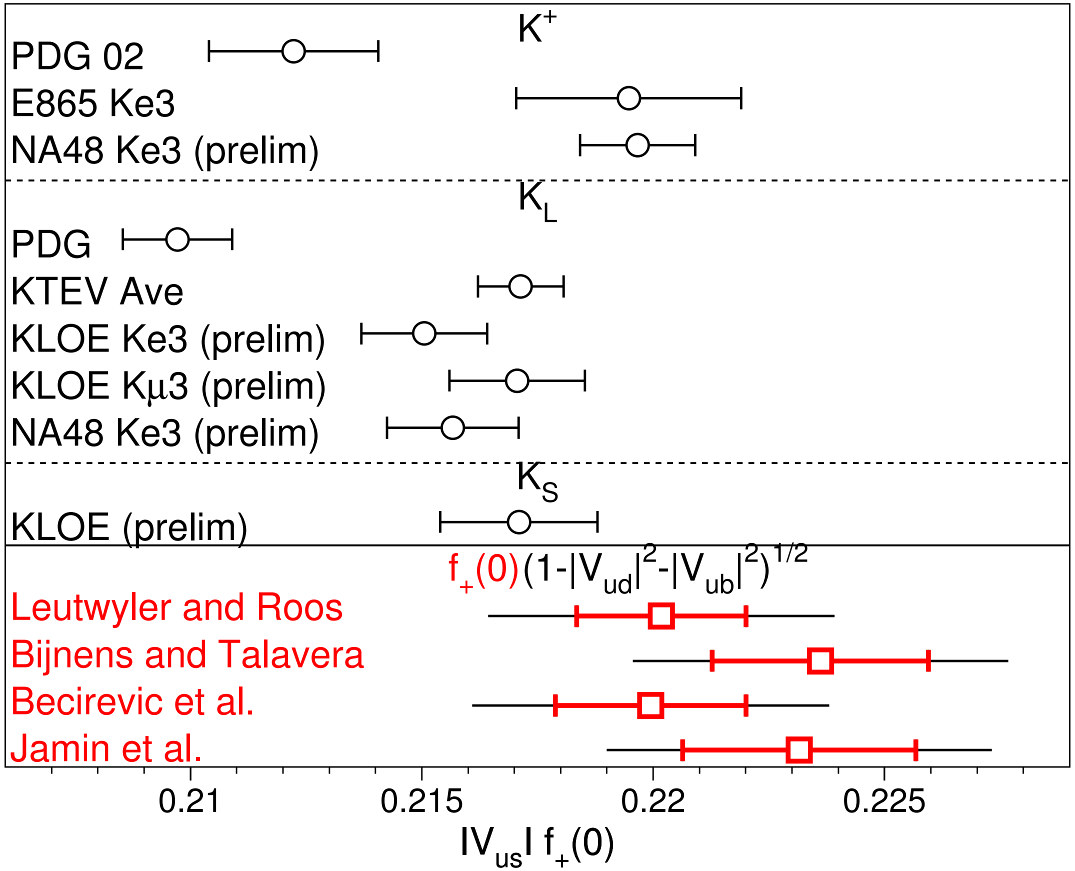
Theory:

$f_+(0)$ is re-calculated by chiral-QCD to p^6 and by lattice QCD. Differ by $\sim 2\%$

$f_{+,0}(t)$ for both chiral and lattice QCD are in agreement with ISTRA+, KTeV measurements.

$V_{us} f_+(0)$ results

$|V_{us}| f_+(0)$ separates theoretical and experimental errors:



(All new K_L results adjusted for the new average lifetime, NA48 is adjusted for the new KTeV form factor measurement).

V_{us}

The final verdict if unitarity problem still exists or not seems to be on the theory side. Using the lattice QCD calculations for $f_+(0)$, and only new experimental data one obtains:

$$|V_{us}| = \begin{cases} 0.2257 \pm 0.0023 & (K_L) \\ 0.2261 \pm 0.0029 & (K_S) \\ 0.2287 \pm 0.0026 & (K_+) \end{cases} \tag{18}$$

Or the average:

$$|V_{us}| = 0.2262 \pm 0.0023$$

With this value for Cabibbo angle, the unitarity:

$$\delta = 1 - \left(|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2\right) = 0.0013 \pm 0.0018$$

is satisfied at 1σ level.

Conclusions

- New experimental data shows large deviation from old PDG average values for both $K_L e3$ and $K^{\pm} e3$ decay rate
- New results resolve longstanding unitarity issue, (if lattice QCD is selected for $f_+(0)$)

Still to do:

- Need final results from KLOE, NA48 $\rightarrow$ may help to resolve $K_L e3$ difference.
- Need new $K^{\pm}$ lifetime measurement (from KLOE) $\rightarrow$ may help to understand $\sim 2.2\sigma$ discrepancy between K_L and $K^{\pm}$ V_{us} determination.

Moral: never leave important measurements un-re-measured